



NINTH EDITION

LINEAR **ALGEBRA** with Applications

GARETH WILLIAMS

NINTH EDITION

LINEAR ALGEBRA

with Applications

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Contents

Preface xi

Part 1 Linear Equations, Vectors, and Matrices 3

1 Linear Equations and Vectors 5

- 1.1** Matrices and Systems of Linear Equations 5
- 1.2** Gauss-Jordan Elimination 17
- 1.3** The Vector Space \mathbf{R}^n 26
- 1.4** Subspaces of \mathbf{R}^n 33
- 1.5** Basis and Dimension 39
- 1.6** Dot Product, Norm, Angle, and Distance 45
(Option: This section can be deferred to just before Section 4.6.)
- *1.7** Curve Fitting, Electrical Networks, and Traffic Flow 57
Chapter 1 Review Exercises 65

2 Matrices and Linear Transformations 69

- 2.1** Addition, Scalar Multiplication, and Multiplication of Matrices 69
- 2.2** Properties of Matrix Operations 80
- 2.3** Symmetric Matrices and Seriation in Archaeology 92
- 2.4** The Inverse of a Matrix and Cryptography 102
- 2.5** Matrix Transformations, Rotations, and Dilations 115
- 2.6** Linear Transformations, Graphics, and Fractals 127
- *2.7** The Leontief Input-Output Model in Economics 137

*Sections marked with an asterisk are optional. The instructor can use these sections to build around the core material to give the course the desired flavor.

***2.8** Markov Chains, Population Movements, and Genetics 143

***2.9** A Communication Model and Group Relationships in Sociology 149

Chapter 2 Review Exercises 159

3 Determinants and Eigenvectors 163

3.1 Introduction to Determinants 163

3.2 Properties of Determinants 171

3.3 Determinants, Matrix Inverses, and Systems of Linear Equations 180

3.4 Eigenvalues and Eigenvectors 188
(Option: Diagonalization of Matrices, Section 5.3, may be discussed at this time.)

***3.5** Google, Demography, Weather Prediction, and Leslie Matrix Models 195

Chapter 3 Review Exercises 208

Part 2 Vector Spaces 211

4 General Vector Spaces 213

4.1 General Vector Spaces and Subspaces 213

4.2 Linear Combinations of Vectors 222

4.3 Linear Independence of Vectors 229

4.4 Properties of Bases 237

4.5 Rank 245

4.6 Projections, Gram-Schmidt Process, and QR Factorization 254

4.7 Orthogonal Complement 266

4.8 Kernel, Range, and the Rank/Nullity Theorem 272

4.9 One-to-One Transformations and Inverse Transformations 284

***4.10** Transformations and Systems of Linear Equations 289

Chapter 4 Review Exercises 296

5 Coordinate Representations 299

5.1 Coordinate Vectors 299

5.2 Matrix Representations of Linear Transformations 307

- 5.3** Diagonalization of Matrices 316
- 5.4** Quadratic Forms, Difference Equations, and Normal Modes 329
- 5.5** Linear Differential Equations (Calculus Prerequisite) 340
- Chapter 5 Review Exercises 346

6 Inner Product Spaces 349

- 6.1** Inner Product Spaces 349
- 6.2** Non-Euclidean Geometry and Special Relativity 358
- 6.3** Approximation of Functions and Coding Theory 363
- 6.4** Least Squares Solutions 370
- Chapter 6 Review Exercises 383

Part 3 Numerical Linear Algebra 385

7 Numerical Methods 387

- 7.1** Gaussian Elimination 387
- 7.2** The Method of LU Decomposition 393
- 7.3** Practical Difficulties in Solving Systems of Equations 400
- 7.4** Iterative Methods for Solving Systems of Linear Equations 409
- 7.5** Eigenvalues by Iteration and Connectivity of Networks 413
- 7.6** The Singular Value Decomposition 423
- Chapter 7 Review Exercises 435

8 Linear Programming 437

- 8.1** A Geometrical Introduction to Linear Programming 437
- 8.2** The Simplex Method 445
- 8.3** Geometrical Explanation of the Simplex Method 451
- Chapter 8 Review Exercises 457

Appendices 459

- A** Cross Product 459
- B** Equations of Planes and Lines in Three-Space 469
- C** Graphing Calculator Manual 477
 - C1** Reduced Echelon Form of a Matrix 477

| | | |
|------------|--|-----|
| C2 | Matrix Operations | 478 |
| C3 | Powers of a Matrix | 478 |
| C4 | Transpose of a Matrix | 479 |
| C5 | Inverse of a Matrix | 479 |
| C6 | Determinant of a Matrix | 479 |
| C7 | Summary of Formats for Row Operations | 480 |
| D | MATLAB Manual | 481 |
| D1 | Entering and Displaying a Matrix (Section 1.1) | 482 |
| D2 | Solving Systems of Linear Equations (Sections 1.1, 1.2, 1.7) | 483 |
| D3 | Dot Product, Norm, Angle, Distance (Section 1.6) | 487 |
| D4 | Matrix Operations (Sections 2.1–2.3) | 488 |
| D5 | Computational Considerations (Section 2.2) | 491 |
| D6 | Inverse of a Matrix (Section 2.4) | 492 |
| D7 | Solving Systems of Equations Using Matrix Inverse (Section 2.4) | 493 |
| D8 | Cryptography (Section 2.4) | 496 |
| D9 | Transformations Defined by Matrices (Sections 2.5, 2.6) | 497 |
| D10 | Fractals (Section 2.6) | 498 |
| D11 | Leontief I/O Model (Section 2.7) | 500 |
| D12 | Markov Chains (Sections 2.8, 3.5) | 502 |
| D13 | Digraphs (Section 2.9) | 504 |
| D14 | Determinants (Sections 3.1–3.3) | 506 |
| D15 | Cramer's Rule (Section 3.3) | 507 |
| D16 | Eigenvalues and Eigenvectors (Sections 3.4, 3.5) | 509 |
| D17 | Linear Combinations, Dependence, Basis, Rank (Sections 1.3, 4.2–4.5) | 511 |
| D18 | Projection, Gram-Schmidt Orthogonalization (Section 4.6) | 514 |
| D19 | QR Factorization (Section 4.6) | 515 |
| D20 | Kernel and Range (Section 4.8) | 518 |
| D21 | Inner Product, Non-Euclidean Geometry (Sections 6.1, 6.2) | 519 |

| | | |
|------------|--|-----|
| D22 | Space-Time Travel (Section 6.2) | 522 |
| D23 | Pseudoinverse and Least Squares Curves (Section 6.4) | 523 |
| D24 | LU Decomposition (Section 7.2) | 527 |
| D25 | Condition Number of a Matrix (Section 7.3) | 529 |
| D26 | Jacobi and Gauss-Seidel Iterative Methods (Section 7.4) | 530 |
| D27 | Singular Value Decomposition (Section 7.6) | 532 |
| D28 | The Simplex Method in Linear Programming (Section 8.2) | 535 |
| D29 | Cross Product (Appendix A) | 536 |
| D30 | MATLAB Commands, Functions, and M-Files | 537 |
| D31 | The Linear Algebra with Applications Toolbox M-Files | 538 |

Answers to Selected Exercises 539

Index 589

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LINEAR ALGEBRA

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KEY FEATURES OF THE NINTH EDITION

- NEW simple-to-advanced organizational framework
- Interesting applications, both theoretical and practical, engage and challenge students
- Carefully explained and illustrated examples highlight key concepts throughout the text
- A MATLAB manual, included as an appendix, consists of 31 sections that tie into course material
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Linear Algebra with Applications, Ninth Edition is designed for the introductory course in linear algebra for students within engineering, mathematics, business management, and physics. Updated to increase clarity and improve student learning, the author provides a flexible blend of theory and engaging applications.

The material in **Linear Algebra with Applications, Ninth Edition** is arranged into three parts that contain core and optional sections:

Part 1 introduces the basics, discussing systems of linear equations, vectors in \mathbf{R}^n matrices, linear transformations, determinants, eigenvalues, and eigenspaces.

Part 2 builds on this material to discuss general vector spaces, and includes such topics as the rank/nullity theorem, inner products, and coordinate representation.

Part 3 completes the course with important ideas and methods in numerical linear algebra including ill-conditioning, pivoting, LU decomposition, and singular value decomposition.

Throughout the text the author provides interesting applications, ranging from theoretical applications such as the use of linear algebra in differential equations, to many practical applications in the fields of electrical engineering, traffic analysis, relativity, history, and more.



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