

# Differential Equations with Boundary-Value Problems

International Metric Edition

Ninth Edition



Dennis G. Zill

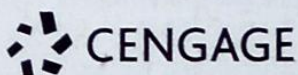
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**Differential Equations with  
Boundary-Value Problems,  
International Metric  
Edition, 9th Edition**

Dennis G. Zill

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