

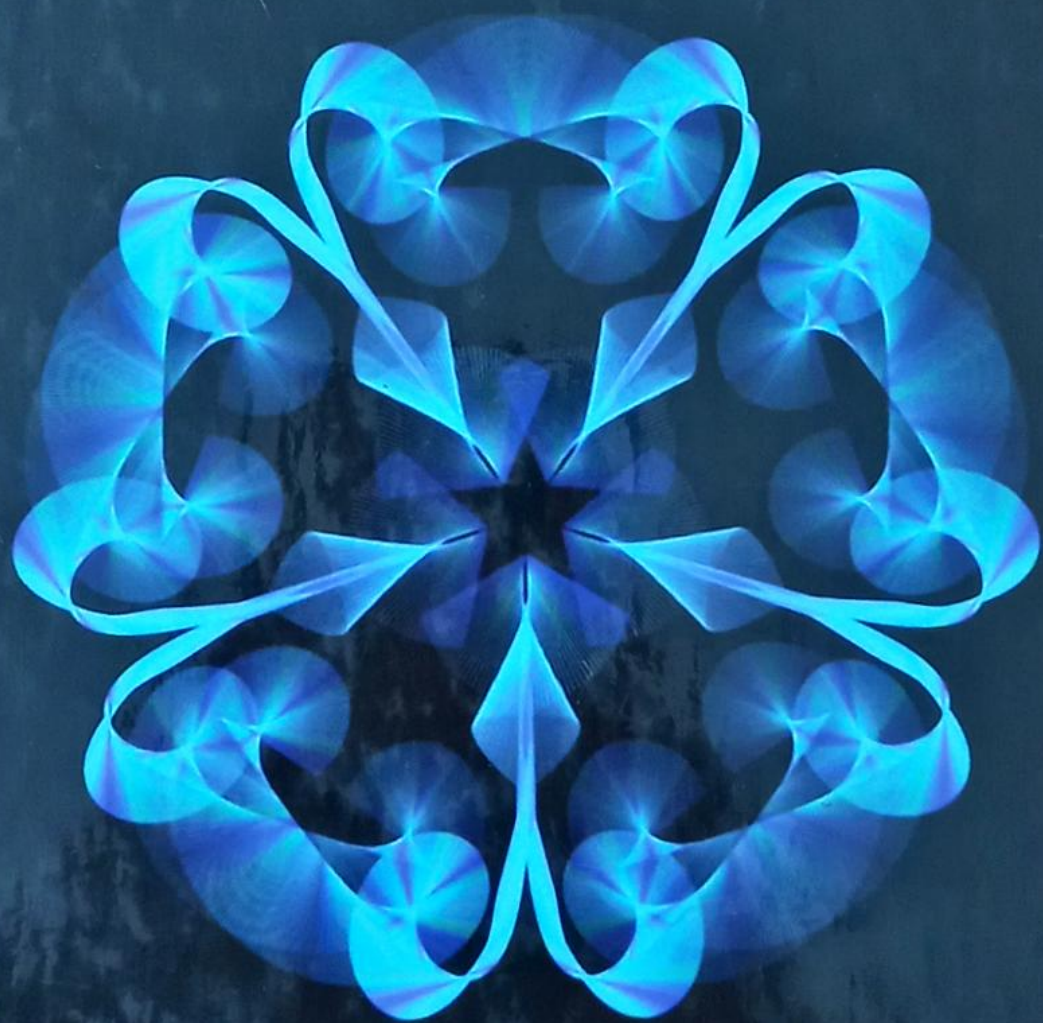
Joseph A. Gallian

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# CONTEMPORARY ABSTRACT ALGEBRA

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Ninth Edition

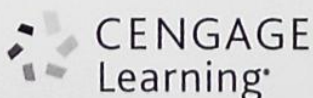


# Contemporary Abstract Algebra

NINTH EDITION

**Joseph A. Gallian**

University of Minnesota Duluth



Australia • Brazil • Mexico • Singapore • United Kingdom • United States



**Contemporary Abstract Algebra,  
Ninth Edition**

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Text Designer: Diane Beasley

Cover Designer: Terri Wright Design

Cover image: Complex Flows by Anne Burns

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WCN: 01-100-101

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Library of Congress Control Number: 2015954307

ISBN: 978-1-305-65796-0

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ISBN-13: 978-1-305-65796-0

ISBN-10: 1-305-65796-9

