



# NONPARAMETRIC STATISTICAL PROCESS CONTROL

---

SUBHABRATA CHAKRABORTI  
MARIEN ALET GRAHAM

---



WILEY

# Nonparametric Statistical Process Control

**Subhabrata Chakraborti**

*University of Alabama  
Tuscaloosa  
USA*

**Marien Alet Graham**

*University of Pretoria  
Pretoria  
South Africa*

**WILEY**

This edition first published 2019  
© 2019 John Wiley & Sons Ltd

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, except as permitted by law. Advice on how to obtain permission to reuse material from this title is available at <http://www.wiley.com/go/permissions>.

The right of Subhabrata Chakraborti and Marien Alet Graham to be identified as the authors of this work has been asserted in accordance with law.

*Registered Offices*

John Wiley & Sons, Inc., 111 River Street, Hoboken, NJ 07030, USA

John Wiley & Sons Ltd, The Atrium, Southern Gate, Chichester, West Sussex, PO19 8SQ, UK

*Editorial Office*

9600 Garsington Road, Oxford, OX4 2DQ, UK

For details of our global editorial offices, customer services, and more information about Wiley products visit us at [www.wiley.com](http://www.wiley.com).

Wiley also publishes its books in a variety of electronic formats and by print-on-demand. Some content that appears in standard print versions of this book may not be available in other formats.

*Limit of Liability/Disclaimer of Warranty*

While the publisher and authors have used their best efforts in preparing this work, they make no representations or warranties with respect to the accuracy or completeness of the contents of this work and specifically disclaim all warranties, including without limitation any implied warranties of merchantability or fitness for a particular purpose. No warranty may be created or extended by sales representatives, written sales materials or promotional statements for this work. The fact that an organization, website, or product is referred to in this work as a citation and/or potential source of further information does not mean that the publisher and authors endorse the information or services the organization, website, or product may provide or recommendations it may make. This work is sold with the understanding that the publisher is not engaged in rendering professional services. The advice and strategies contained herein may not be suitable for your situation. You should consult with a specialist where appropriate. Further, readers should be aware that websites listed in this work may have changed or disappeared between when this work was written and when it is read. Neither the publisher nor authors shall be liable for any loss of profit or any other commercial damages, including but not limited to special, incidental, consequential, or other damages.

*Library of Congress Cataloging-in-Publication Data*

Names: Chakraborti, Subhabrata, author. | Graham, Marien Alet, author.

Title: Nonparametric statistical process control / Subhabrata Chakraborti, Marien Alet Graham.

Description: Hoboken, NJ : John Wiley & Sons, 2019. | Includes bibliographical references and index. |

Identifiers: LCCN 2018023685 (print) | LCCN 2018032639 (ebook) | ISBN 9781118890677 (Adobe PDF) | ISBN 9781118890578 (ePub) | ISBN 9781118456033 (hardcover)

Subjects: LCSH: Nonparametric statistics. | Process control—Statistical methods.

Classification: LCC QA278.8 (ebook) | LCC QA278.8 .C445 2018 (print) | DDC 519.5—dc23

LC record available at <https://lcn.loc.gov/2018023685>

Cover Design: Wiley

Cover Image: © Rachelle Burnside/Shutterstock

Set in 11/13pt CMR10 by SPi Global, Chennai, India

Printed in Singapore by C.O.S. Printers Pte Ltd

10 9 8 7 6 5 4 3 2 1

# Contents

About the Authors	xiii
Preface	xv
About the companion website	xix
<b>1 Background/Review of Statistical Concepts</b>	<b>1</b>
Chapter Overview	1
1.1 Basic Probability	1
1.2 Random Variables and Their Distributions	3
1.3 Random Sample	12
1.4 Statistical Inference	16
1.5 Role of the Computer	22
<b>2 Basics of Statistical Process Control</b>	<b>23</b>
Chapter Overview	23
2.1 Basic Concepts	23
2.1.1 Types of Variability	23
2.1.2 The Control Chart	25
2.1.3 Construction of Control Charts	29
2.1.4 Variables and Attributes Control Charts	30
2.1.5 Sample Size or Subgroup Size	31
2.1.6 Rational Subgrouping	31
2.1.7 Nonparametric or Distribution-free	34
2.1.8 Monitoring Process Location and/or Process Scale	36
2.1.9 Case K and Case U	37
2.1.10 Control Charts and Hypothesis Testing	37
2.1.11 General Steps in Designing a Control Chart	39
2.1.12 Measures of Control Chart Performance	39
2.1.12.1 False Alarm Probability ( <i>FAP</i> )	41
2.1.12.2 False Alarm Rate ( <i>FAR</i> )	43
2.1.12.3 The Average Run-length ( <i>ARL</i> )	43

2.1.12.4	Standard Deviation of Run-length ( <i>SDRL</i> ) . . . . .	44
2.1.12.5	Percentiles of Run-length . . . . .	44
2.1.12.6	Average Number of Samples to Signal ( <i>ANSS</i> ) . . . . .	48
2.1.12.7	Average Number of Observations to Signal ( <i>ANOS</i> ) . . . . .	48
2.1.12.8	Average Time to Signal ( <i>ATS</i> ) . . . . .	48
2.1.12.9	Number of Individual Items Inspected ( <i>I</i> ) . . . . .	49
2.1.13	Operating Characteristic Curves ( <i>OC</i> -curves) . . . . .	50
2.1.14	Design of Control Charts . . . . .	51
2.1.14.1	Sample Size, Sampling Frequency, and Variable Sample Sizes . . . . .	51
2.1.14.2	Variable Control Limits . . . . .	54
2.1.14.3	Standardized Control Limits . . . . .	56
2.1.15	Size of a Shift . . . . .	57
2.1.16	Choice of Control Limits . . . . .	59
2.1.16.1	$k$ -sigma Limits . . . . .	59
2.1.16.2	Probability Limits . . . . .	60
<b>3</b>	<b>Parametric Univariate Variables Control Charts</b> . . . . .	<b>63</b>
	Chapter Overview . . . . .	63
3.1	Introduction . . . . .	64
3.2	Parametric Variables Control Charts in Case K . . . . .	64
3.2.1	Shewhart Control Charts . . . . .	65
3.2.2	CUSUM Control Charts . . . . .	67
3.2.3	EWMA Control Charts . . . . .	72
3.3	Types of Parametric Variables Charts in Case K: Illustrative Examples . . . . .	77
3.3.1	Shewhart Control Charts . . . . .	77
3.3.1.1	Shewhart Control Charts for Monitoring Process Mean . . . . .	77
3.3.1.2	Shewhart Control Charts for Monitoring Process Variation . . . . .	79
3.3.2	CUSUM Control Charts . . . . .	84
3.3.3	EWMA Control Charts . . . . .	87
3.4	Shewhart, EWMA, and CUSUM Charts: Which to Use When . . . . .	90
3.5	Control Chart Enhancements . . . . .	91
3.5.1	Sensitivity Rules . . . . .	91
3.5.2	Runs-type Signaling Rules . . . . .	95
3.5.2.1	Signaling Indicators . . . . .	97

3.6	Run-length Distribution in the Specified Parameter Case (Case K) . . . . .	110
3.6.1	Methods of Calculating the Run-length Distribution . . . . .	110
3.6.1.1	The Exact Approach (for Shewhart and some Shewhart-type Charts) . . . . .	110
3.6.1.2	The Markov Chain Approach . . . . .	111
3.6.1.3	The Integral Equation Approach . . . . .	128
3.6.1.4	The Computer Simulations (the Monte Carlo) Approach . . . . .	128
3.7	Parameter Estimation Problem and Its Effects on the Control Chart Performance . . . . .	131
3.8	Parametric Variables Control Charts in Case U . . . . .	133
3.8.1	Shewhart Control Charts in Case U . . . . .	133
3.8.1.1	Shewhart Control Charts for the Mean in Case U . . . . .	133
3.8.1.2	Shewhart Control Charts for the Standard Deviation in Case U . . . . .	134
3.8.2	CUSUM Chart for the Mean in Case U . . . . .	137
3.8.3	EWMA Chart for the Mean in Case U . . . . .	137
3.9	Types of Parametric Control Charts in Case U: Illustrative Examples . . . . .	138
3.9.1	Charts for the Mean . . . . .	138
3.9.2	Charts for the Standard Deviation . . . . .	141
3.9.2.1	Using the Estimator $S_p$ . . . . .	144
3.10	Run-length Distribution in the unknown Parameter Case (Case U) . . . . .	153
3.10.1	Methods of Calculating the Run-length Distribution and Its Properties: The Conditioning/Unconditioning Method . . . . .	153
3.10.1.1	The Shewhart Chart for the Mean in Case U . . . . .	153
3.10.1.2	The Shewhart Chart for the Variance in Case U . . . . .	169
3.10.1.3	The CUSUM Chart for the Mean in Case U . . . . .	170
3.10.1.4	The EWMA Chart for the Mean in Case U . . . . .	171
3.11	Control Chart Enhancements . . . . .	172
3.11.1	Run-length Calculation for Runs-type Signaling Rules in Case U . . . . .	172
3.12	Phase I Control Charts . . . . .	174
3.12.1	Phase I $\bar{X}$ -chart . . . . .	174
3.13	Size of Phase I Data . . . . .	176

# CONTENTS

x

3.14 Robustness of Parametric Control Charts	177
Appendix 3.1 Some Derivations for the EWMA Control Chart	178
Appendix 3.2 Markov Chains	180
Appendix 3.3 Some Derivations for the Shewhart Dispersion Charts	184
<b>4 Nonparametric (Distribution-free) Univariate Variables Control Charts</b>	<b>187</b>
Chapter Overview	187
4.1 Introduction	187
4.2 Distribution-free Variables Control Charts in Case K	189
4.2.1 Shewhart Control Charts	189
4.2.1.1 Shewhart Control Charts Based on Signs	189
4.2.1.2 Shewhart Control Charts Based on Signed-ranks	196
4.2.2 CUSUM Control Charts	202
4.2.2.1 CUSUM Control Charts Based on Signs	202
4.2.2.2 A CUSUM Sign Control Chart with Runs-type Signaling Rules	203
4.2.2.3 Methods of Calculating the Run-length Distribution	203
4.2.2.4 CUSUM Control Charts Based on Signed-ranks	205
4.2.3 EWMA Control Charts	208
4.2.3.1 EWMA Control Charts Based on Signs	208
4.2.3.2 EWMA Control Charts Based on Signs with Runs-type Signaling Rules	210
4.2.3.3 Methods of Calculating the Run-length Distribution	210
4.2.3.4 EWMA Control Charts Based on Signed-ranks	214
4.2.3.5 An EWMA-SR control chart with runs-type signaling rules	216
4.2.3.6 Methods of Calculating the Run-length Distribution	216
4.3 Distribution-free Control Charts in Case K: Illustrative Examples	219
4.3.1 Shewhart Control Charts	219
4.3.2 CUSUM Control Charts	229
4.3.3 EWMA Control Charts	243
4.4 Distribution-free Variables Control Charts in Case U	253

# CONTENTS

xi

4.4.1 Shewhart Control Charts	254
4.4.1.1 Shewhart Control Charts Based on the Precedence Statistic	254
4.4.1.2 Shewhart Control Charts Based on the Mann-Whitney Test Statistic	275
4.4.2 CUSUM Control Charts	281
4.4.2.1 CUSUM Control Charts Based on the Exceedance Statistic	281
4.4.2.2 CUSUM Control Charts Based on the Wilcoxon Rank-sum Statistic	285
4.4.3 EWMA Control Charts	287
4.4.3.1 EWMA Control Charts Based on the Exceedance Statistic	287
4.4.3.2 EWMA Control Charts Based on the Wilcoxon Rank-sum Statistic	290
4.5 Distribution-free Control Charts in Case U: Illustrative Examples	293
4.5.1 Shewhart Control Charts	293
4.5.2 CUSUM Control Charts	295
4.5.3 EWMA Control Charts	302
4.6 Effects of Parameter Estimation	307
4.7 Size of Phase I Data	307
4.8 Control Chart Enhancements	308
4.8.1 Sensitivity and Runs-type Signaling Rules	308
Appendix 4.1 Shewhart Control Charts	311
Appendix 4.1.1 The Shewhart-Prec Control Chart	311
Appendix 4.2 CUSUM Control Charts	312
Appendix 4.2.1 The CUSUM-EX Control Chart	312
Appendix 4.2.2 The CUSUM-rank Control Chart	314
Appendix 4.3 EWMA Control Charts	317
Appendix 4.3.1 The EWMA-SN Control Chart	317
Appendix 4.3.2 The EWMA-SR Control Chart	318
Appendix 4.3.3 The EWMA-EX Control Chart	319
Appendix 4.3.4 The EWMA-rank Control Chart	323
<b>5 Miscellaneous Univariate Distribution-free (Nonparametric) Variables Control Charts</b>	<b>325</b>
Chapter Overview	325
5.1 Introduction	325
5.2 Other Univariate Distribution-free (Nonparametric) Variables Control Charts	326
5.2.1 Phase I Control Charts	326
5.2.1.1 Introduction	326
5.2.1.2 Phase I Control Charts for Location	331

5.2.2	Special Cases of Precedence Charts . . . . .	343
5.2.2.1	The Min Chart . . . . .	343
5.2.2.2	The CUMIN Chart . . . . .	346
5.2.3	Control Charts Based on Bootstrapping . . . . .	348
5.2.3.1	Methodology . . . . .	351
5.2.4	Change-point Models . . . . .	353
5.2.5	Some Adaptive Charts . . . . .	357
5.2.5.1	Introduction . . . . .	357
5.2.5.2	Variable Sampling Interval (VSI) and Variable Sample Size (VSS) Charts . . .	358
5.2.5.3	Other Adaptive Schemes . . . . .	359
5.2.5.4	Properties and Performance Measures of Adaptive Charts . . . . .	360
5.2.5.5	Adaptive Nonparametric Control Charts . . . . .	362
<b>Appendix A</b>	<b>Tables</b> . . . . .	<b>369</b>
<b>Appendix B</b>	<b>Programmes</b> . . . . .	<b>381</b>
<b>References</b>		<b>413</b>
<b>Index</b>		<b>425</b>

# A UNIQUE APPROACH TO UNDERSTANDING THE FOUNDATIONS OF STATISTICAL QUALITY CONTROL WITH A FOCUS ON NONPARAMETRIC CONTROL CHARTING METHODOLOGIES

Statistical Process Control (SPC) methods have a long and successful history in industrial statistics and have revolutionized many facets of industrial production around the world. This book addresses recent developments in SPC bringing the modern use of computers and simulations along with the theory within the reach of both the researchers and practitioners. The emphasis is on the burgeoning field of nonparametric SPC (NSPC) and the many new methodologies developed by researchers worldwide. Over the last several years, research in SPC, particularly on control charts, has seen phenomenal growth. Control charts are no longer confined to manufacturing and are now applied for process control and monitoring in a wide array of applications, from education, to environmental monitoring, to disease mapping, to crime prevention. This book treats quality control methodology, especially control charts, from a statistician's viewpoint, striking a careful balance between theory and practice. Although the focus is on the newer nonparametric control charts, the reader is first introduced to the main classes of the parametric control charts and the associated theory, so that the proper foundational background can be laid.

Offering a uniquely balanced presentation of both theory and practice, *Nonparametric Statistical Process Control* is a vital resource for students, interested practitioners, researchers, and anyone with an appropriate background in statistics interested in learning about the foundations of SPC and latest developments in NSPC.


**SUBHABRATA CHAKRABORTI, PhD** is Professor of Statistics and Morrow Faculty Excellence Fellow at the University of Alabama, Tuscaloosa, AL, USA. He is a Fellow of the American Statistical Association and an elected member of the International Statistical Institute. Professor Chakraborti has contributed in a number of research areas, including censored data analysis and income inference. His current research interests include development of statistical methods in general and nonparametric methods in particular for statistical process control. He has been a Fulbright Senior Scholar to South Africa and a visiting professor in several countries, including India, Holland and Brazil. Cited for his mentoring and collaborative work with students and scholars from around the world, Professor Chakraborti has presented seminars, delivered keynote/plenary addresses and conducted research workshops at various conferences.

**MARIEN ALET GRAHAM, PhD** is a senior lecturer at the Department of Science, Mathematics and Technology Education at the University of Pretoria, Pretoria, South Africa. She holds an Y1 rating from the South African National Research Foundation (NRF). Her current research interests are in Statistical Process Control, Nonparametric Statistics and Statistical Education. She has published several articles in international peer review journals and presented her work at various conferences.

Cover Design: Wiley  
Cover Image: © Rachelle Burnside/  
Shutterstock

[www.wiley.com/go/chakraborti/](http://www.wiley.com/go/chakraborti/)  
Nonparametric\_Statistical\_Process\_Control



 Also available  
as an e-book

ISBN 978-1-118-45603-3



9 781118 456033

**WILEY**