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# Algebra II Essentials

for  
**dummies**<sup>®</sup>

A Wiley Brand



The “must-know”  
formulas and equations

What you need to know  
to master Algebra II

Core topics in quick,  
focused lessons

**Mary Jane Sterling**

Emerita Instructor  
of Mathematics



# Algebra II Essentials

by Mary Jane Sterling

for  
**dummies®**  
A Wiley Brand

## **Algebra II Essentials For Dummies®**

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# Your ticket to scoring high in Algebra II

*Algebra II Essentials For Dummies* sticks to the point, with concise explanations of critical concepts taught in a typical Algebra II course. Get the lowdown on algebraic properties, exponential rules, and factoring techniques. Discover how to use exponential and logarithmic functions to solve algebraic problems. Learn how to work with rational and radical equations. It's all here in this easy-to-read guide that's perfect for cramming, homework help, or as a reference for parents helping students prepare for exams.

## Inside...

- How to work with exponents
- Clear steps for solving linear equations
- The quadratic formula
- Graphing techniques
- Tips for finding the y-intercept
- Help with solving systems of equations
- Formulas you need to know

**Mary Jane Sterling** taught algebra, business calculus, geometry, and finite mathematics at Bradley University in Peoria, Illinois, for more than 30 years. She is the author of *Algebra II For Dummies* and *Algebra II Workbook For Dummies*.

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