



# Elementary Linear Algebra

Eighth Edition

Ron Larson

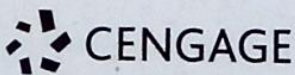


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Australia • Brazil • Mexico • Singapore • United Kingdom • United States



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**8th Edition**  
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