

# CALCULUS

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**11e**



**RON LARSON**

**BRUCE EDWARDS**



# CALCULUS

with **CalcChat**® and **CalcView**®

**11e**

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**Calculus, Eleventh Edition**  
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# Contents

<b>P</b>	<b>▷ Preparation for Calculus</b>	<b>1</b>
P.1	Graphs and Models	2
P.2	Linear Models and Rates of Change	10
P.3	Functions and Their Graphs	19
P.4	Review of Trigonometric Functions	31
	<b>Review Exercises</b>	41
	<b>P.S. Problem Solving</b>	43
<b>1</b>	<b>▷ Limits and Their Properties</b>	<b>45</b>
1.1	A Preview of Calculus	46
1.2	Finding Limits Graphically and Numerically	52
1.3	Evaluating Limits Analytically	63
1.4	Continuity and One-Sided Limits	74
1.5	Infinite Limits	87
	<b>Section Project: Graphs and Limits of Trigonometric Functions</b>	94
	<b>Review Exercises</b>	95
	<b>P.S. Problem Solving</b>	97
<b>2</b>	<b>▷ Differentiation</b>	<b>99</b>
2.1	The Derivative and the Tangent Line Problem	100
2.2	Basic Differentiation Rules and Rates of Change	110
2.3	Product and Quotient Rules and Higher-Order Derivatives	122
2.4	The Chain Rule	133
2.5	Implicit Differentiation	144
	<b>Section Project: Optical Illusions</b>	151
2.6	Related Rates	152
	<b>Review Exercises</b>	161
	<b>P.S. Problem Solving</b>	163
<b>3</b>	<b>▷ Applications of Differentiation</b>	<b>165</b>
3.1	Extrema on an Interval	166
3.2	Rolle's Theorem and the Mean Value Theorem	174
3.3	Increasing and Decreasing Functions and the First Derivative Test	181
	<b>Section Project: Even Fourth-Degree Polynomials</b>	190
3.4	Concavity and the Second Derivative Test	191
3.5	Limits at Infinity	199
3.6	A Summary of Curve Sketching	209
3.7	Optimization Problems	219
	<b>Section Project: Minimum Time</b>	228
3.8	Newton's Method	229
3.9	Differentials	235
	<b>Review Exercises</b>	242
	<b>P.S. Problem Solving</b>	245



<b>4</b>	<b>▷ Integration</b>	<b>247</b>
4.1	Antiderivatives and Indefinite Integration	248
4.2	Area	258
4.3	Riemann Sums and Definite Integrals	270
4.4	The Fundamental Theorem of Calculus	281
	Section Project: Demonstrating the Fundamental Theorem	295
4.5	Integration by Substitution	296
	Review Exercises	309
	P.S. Problem Solving	311
<b>5</b>	<b>▷ Logarithmic, Exponential, and Other Transcendental Functions</b>	<b>313</b>
5.1	The Natural Logarithmic Function: Differentiation	314
5.2	The Natural Logarithmic Function: Integration	324
5.3	Inverse Functions	333
5.4	Exponential Functions: Differentiation and Integration	342
5.5	Bases Other than $e$ and Applications	352
	Section Project: Using Graphing Utilities to Estimate Slope	361
5.6	Indeterminate Forms and L'Hôpital's Rule	362
5.7	Inverse Trigonometric Functions: Differentiation	373
5.8	Inverse Trigonometric Functions: Integration	382
5.9	Hyperbolic Functions	390
	Section Project: Mercator Map	399
	Review Exercises	400
	P.S. Problem Solving	403
<b>6</b>	<b>▷ Differential Equations</b>	<b>405</b>
6.1	Slope Fields and Euler's Method	406
6.2	Growth and Decay	415
6.3	Separation of Variables and the Logistic Equation	423
6.4	First-Order Linear Differential Equations	432
	Section Project: Weight Loss	438
	Review Exercises	439
	P.S. Problem Solving	441
<b>7</b>	<b>▷ Applications of Integration</b>	<b>443</b>
7.1	Area of a Region Between Two Curves	444
7.2	Volume: The Disk Method	454
7.3	Volume: The Shell Method	465
	Section Project: Saturn	473
7.4	Arc Length and Surfaces of Revolution	474
7.5	Work	485
	Section Project: Pyramid of Khufu	493
7.6	Moments, Centers of Mass, and Centroids	494
7.7	Fluid Pressure and Fluid Force	505
	Review Exercises	511
	P.S. Problem Solving	513

<b>8</b>	<b>▷</b>	<b>Integration Techniques and Improper Integrals</b>	<b>515</b>
8.1		Basic Integration Rules	516
8.2		Integration by Parts	523
8.3		Trigonometric Integrals	532
		<b>Section Project: The Wallis Product</b>	540
8.4		Trigonometric Substitution	541
8.5		Partial Fractions	550
8.6		Numerical Integration	559
8.7		Integration by Tables and Other Integration Techniques	566
8.8		Improper Integrals	572
		<b>Review Exercises</b>	583
		<b>P.S. Problem Solving</b>	585
<b>9</b>	<b>▷</b>	<b>Infinite Series</b>	<b>587</b>
9.1		Sequences	588
9.2		Series and Convergence	599
		<b>Section Project: Cantor's Disappearing Table</b>	608
9.3		The Integral Test and $p$ -Series	609
		<b>Section Project: The Harmonic Series</b>	615
9.4		Comparisons of Series	616
9.5		Alternating Series	623
9.6		The Ratio and Root Tests	631
9.7		Taylor Polynomials and Approximations	640
9.8		Power Series	651
9.9		Representation of Functions by Power Series	661
9.10		Taylor and Maclaurin Series	668
		<b>Review Exercises</b>	680
		<b>P.S. Problem Solving</b>	683
<b>10</b>	<b>▷</b>	<b>Conics, Parametric Equations, and Polar Coordinates</b>	<b>685</b>
10.1		Conics and Calculus	686
10.2		Plane Curves and Parametric Equations	700
		<b>Section Project: Cycloids</b>	709
10.3		Parametric Equations and Calculus	710
10.4		Polar Coordinates and Polar Graphs	719
		<b>Section Project: Cassini Oval</b>	728
10.5		Area and Arc Length in Polar Coordinates	729
10.6		Polar Equations of Conics and Kepler's Laws	738
		<b>Review Exercises</b>	746
		<b>P.S. Problem Solving</b>	749



<b>11</b>	<b>▷ Vectors and the Geometry of Space</b>	<b>751</b>
11.1	Vectors in the Plane 752	
11.2	Space Coordinates and Vectors in Space 762	
11.3	The Dot Product of Two Vectors 770	
11.4	The Cross Product of Two Vectors in Space 779	
11.5	Lines and Planes in Space 787	
	<b>Section Project: Distances in Space</b> 797	
11.6	Surfaces in Space 798	
11.7	Cylindrical and Spherical Coordinates 808	
	<b>Review Exercises</b> 815	
	<b>P.S. Problem Solving</b> 817	
<b>12</b>	<b>▷ Vector-Valued Functions</b>	<b>819</b>
12.1	Vector-Valued Functions 820	
	<b>Section Project: Witch of Agnesi</b> 827	
12.2	Differentiation and Integration of Vector-Valued Functions 828	
12.3	Velocity and Acceleration 836	
12.4	Tangent Vectors and Normal Vectors 845	
12.5	Arc Length and Curvature 855	
	<b>Review Exercises</b> 867	
	<b>P.S. Problem Solving</b> 869	
<b>13</b>	<b>▷ Functions of Several Variables</b>	<b>871</b>
13.1	Introduction to Functions of Several Variables 872	
13.2	Limits and Continuity 884	
13.3	Partial Derivatives 894	
13.4	Differentials 904	
13.5	Chain Rules for Functions of Several Variables 911	
13.6	Directional Derivatives and Gradients 919	
13.7	Tangent Planes and Normal Lines 931	
	<b>Section Project: Wildflowers</b> 939	
13.8	Extrema of Functions of Two Variables 940	
13.9	Applications of Extrema 948	
	<b>Section Project: Building a Pipeline</b> 955	
13.10	Lagrange Multipliers 956	
	<b>Review Exercises</b> 964	
	<b>P.S. Problem Solving</b> 967	
<b>14</b>	<b>▷ Multiple Integration</b>	<b>969</b>
14.1	Iterated Integrals and Area in the Plane 970	
14.2	Double Integrals and Volume 978	
14.3	Change of Variables: Polar Coordinates 990	
14.4	Center of Mass and Moments of Inertia 998	
	<b>Section Project: Center of Pressure on a Sail</b> 1005	
14.5	Surface Area 1006	
	<b>Section Project: Surface Area in Polar Coordinates</b> 1012	
14.6	Triple Integrals and Applications 1013	
14.7	Triple Integrals in Other Coordinates 1024	
	<b>Section Project: Wrinkled and Bumpy Spheres</b> 1030	
14.8	Change of Variables: Jacobians 1031	
	<b>Review Exercises</b> 1038	
	<b>P.S. Problem Solving</b> 1041	

**15 ▷ Vector Analysis 1043**

- 15.1 Vector Fields 1044
- 15.2 Line Integrals 1055
- 15.3 Conservative Vector Fields and Independence of Path 1069
- 15.4 Green's Theorem 1079
  - Section Project: Hyperbolic and Trigonometric Functions** 1087
- 15.5 Parametric Surfaces 1088
- 15.6 Surface Integrals 1098
  - Section Project: Hyperboloid of One Sheet** 1109
- 15.7 Divergence Theorem 1110
- 15.8 Stokes's Theorem 1118
  - Review Exercises** 1124
  - P.S. Problem Solving** 1127

**16 ▷ Additional Topics in Differential Equations (Online)\***

- 16.1 Exact First-Order Equations
- 16.2 Second-Order Homogeneous Linear Equations
- 16.3 Second-Order Nonhomogeneous Linear Equations
  - Section Project: Parachute Jump**
- 16.4 Series Solutions of Differential Equations
  - Review Exercises**
  - P.S. Problem Solving**

**Appendices**

- Appendix A: Proofs of Selected Theorems A2
- Appendix B: Integration Tables A3
- Appendix C: Precalculus Review (Online)\*
- Appendix D: Rotation and the General Second-Degree Equation (Online)\*
- Appendix E: Complex Numbers (Online)\*
- Appendix F: Business and Economic Applications (Online)\*
- Appendix G: Fitting Models to Data (Online)\*

Answers to All Odd-Numbered Exercises A7  
Index A121

\*Available at the text-specific website [www.cengagebrain.com](http://www.cengagebrain.com)



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